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A Theorem on Asymptotic Expansion of Feynman Amplitudes

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Abstract. For any Feynman amplitude, where any subset of invariants and/or squared masses is scaled by a real parameter λ going to zero or infinity, the existence of an expansion in powers of λ and $\ln\lambda$ is proved, and a method is given for determining such an expansion. This is shown quite generally in euclidean metric, whatever the external momenta (exceptional or not) and the internal masses (vanishing or not) may be, and for some simple cases in minkowskian metric, assuming only finiteness of the – eventually renormalized – amplitude before scaling. The method uses what is called "Multiple Mellin representation", the validity of which is related to a "generalized power-counting" theorem.

I. Introduction

In this paper we give a mathematical method for studying the asymptotic behaviour of Feynman amplitudes, that is integrals $G(a_k)$ corresponding to given Feynman graphs. Notation $\{a_k\}$ represents the set of invariants $(\sum p)^2$ built from external momenta p, and of internal squared masses m_i^2. By asymptotic behaviour we mean an expansion in a real parameter λ scaling some a_k,s, say $\{a_m\}$, the other ones, say $\{a_n\}$, remaining fixed. Conventionally we take λ as going to infinity, but of course the method applies as well to the case of invariants or masses going to zero, by dimensional argument:

$$G\left(a_m, \frac{1}{\lambda}a_n\right) = \lambda^{\omega} G(\lambda a_m, a_n). \tag{I.1}$$

We emphasize that our method is quite general, since it applies to any asymptotic limit (any choice of the subset $\{a_m\}$), for arbitrarily given external momenta, generic or exceptional, and for arbitrary finite or vanishing masses. In this paper we consider for simplicity only scalar particles with non-derivative couplings. Then for any Feynman amplitude in Euclidean metric (and for some

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