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# Limit Gibbs State for Some Classes of One-Dimensional Systems of Quantum Statistical Mechanics*

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**Abstract.** We prove the existence of the limit Gibbs state for one-dimensional continuous quantum fermion systems with non-hard-core, non-negative, rapidly decreasing pair interaction potentials. Existence of the limit Gibbs state is also established for one-dimensional continuous quantum boson systems with pair interaction potentials as above which, in addition, increase sufficiently fast at small distances.

## 0. Introduction

The mathematically rigorous theory of phase transitions in systems with infinitely many degrees of freedom [1–3] is now developped in the main for the class of spin systems (see, e.g., [4, 5] and references there). The case of continuous systems, particularly, of continuous quantum systems (c.q.s.) is more difficult. Even fundamental physical notions such as those of time evolution and equilibrium states have yet no satisfactory definitions, except a number of exactly solvable models [3, 6]. One of principal difficulties is that the Hamiltonians of c.q.s. in bounded domains are unbounded operators.

The main method for studying the c.q.s. is now the Wiener integral representation based on the Feynman-Kac formula and used first by Ginibre [7]. This method allows one to exploit an analogy between classical and quantum systems and to investigate dilute quantum gases [7] and one-dimensional c.q.s. [8]. In particular, in [8] we considered the case of one-dimensional c.q.s. with a hard-core, long-range interaction between particles.

The present paper deals with several types of one-dimensional c.q.s. where the particles interact via a non-hard-core long-range interaction potential which is non-negative and rapidly decreasing. We establish here the existence of the limit Gibbs state in the infinite volume. Its properties such as regularity (cluster

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