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Pointwise Bounds for Schrödinger Eigenstates

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Abstract. In a different paper we constructed imaginary time Schrödinger operators $H_q = -\frac{1}{2}\Delta + V$ acting on $L^q(\mathbb{R}^n, dx)$. The negative part of typical potential function V was assumed to be in $L^\infty + L^q$ for some $p > \max\{1, n/2\}$. Our proofs were based on the evaluation of Kac's averages over Brownian motion paths. The present paper continues this study: using probabilistic techniques we prove pointwise upper bounds for L^q-Schrödinger eigenstates and pointwise lower bounds for the corresponding groundstate. The potential functions V are assumed to be neither smooth nor bounded below. Consequently, our results generalize Schnol's and Simon's ones. Moreover probabilistic proofs seem to be shorter and more informative than existing ones.

I. Introduction

The study of Schrödinger operator via probabilistic techniques originates in the famous work of Kac [14]. Using Wiener's measure in the case of heat equation, he made mathematically rigorous the heuristic prescriptions given by Feynman to solve Schrödinger equation. Stimulated by Kac's paper, something of an industry has developed, and the probabilistic approach is by now standard. To-day it is clear that Brownian motion averages are a good device to define and study the diffusion semigroup for fairly general potential functions V, and to prove the negative infinitesimal generator is a suitable extension of the formal differential operator $-\frac{1}{2}\Delta + V$. See for example [18, 10, 1, 11, 5, 16 and 4]. Let us note that in most of these papers V was assumed to be bounded below. Thanks to [2, Théorème 1] or [4, Theorem 2.1] this restrictive hypothesis can be weakened. Moreover nice spectral properties have been proved by means of similar techniques (see for example [9, 15, 2, and 4]).

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