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Peierls Condition and Number of Ground States

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Abstract. Recently Pirogov and Sinai developped a theory of phase transitions in systems satisfying Peierls condition. We give a criterium for the Peierls condition to hold and apply it to several systems. In particular we prove that ferromagnetic system satisfies the Peierls condition iff its (internal) symmetry group is finite. And using an algebraic argument we show that in two dimensions the symmetry groups of reduced translation invariant systems is finite.

Introduction

A theory of phase transitions at low temperatures in general classical lattice systems was developped recently by Pirogov and Sinai [8, 9]. It applies to systems satisfying the Peierls condition of [8, 4, 9]. In particular, finitness of the number of (periodic) ground states is assumed.

The main results of this paper are: Criterium for Peierls condition to hold (Section 3), *a necessary and sufficient condition for ferromagnetic systems to* satisfy Peierls condition (Sections 10 and 11) and a theorem on finitness of the number of ground states of ferromagnetic systems in two dimensions (Section 12). In addition, to demonstrate how the Criterium works, we apply it to several models. The criterium applies to all models we consider and often yields more complete picture than previously obtained.

In Sections 1 and 2 we introduce the framework and recall the Peierls condition. In Section 3 we formulate and prove Criterium; in the proof we take advantage of the flexibility of the Gerzik-Debrushin [5, 4, 8, 9] definition of contours which allows us to replace previous considerations based on special symmetries of models by a general compactness argument. After preparatory remarks of Section 4, in Sections 5–9 we apply Criterium to several models; the discussion of the antiferromagnet in Section 5 which leads to well known results of Dobrushin [3] is especially detailed.

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