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The High Density Limit for Lattice Spin Models

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Abstract. The n-vector, spherical and quantum spin models are considered on a regular lattice with co-ordination number q. In the limit $q\to\infty$ it is proved algebraically that the free energies are given by the corresponding Curie-Weiss or mean-field expressions.

1. Introduction

The classical Curie-Weiss theory of ferromagnetism has been established rigorously by considering various limiting lattice spin systems. It is known, for example (Thompson and Silver [1], Pearce and Thompson [2]), that if the spins interact with a Kac-type pair potential

$$\varrho_{\boldsymbol{i}\boldsymbol{j}}=\gamma^d\varrho(\gamma|\boldsymbol{i}-\boldsymbol{j}|), \tag{1.1}$$

the Curie-Weiss theory results in the long-range limit $\gamma\to 0$. In addition it has been shown (Pearce and Thompson [3]) that the Curie-Weiss theory arises for a system of n-vector spins, interacting with extreme anisotropy, in the spherical or infinite spin-dimensionality limit ($n\to\infty$).

In this paper we will be concerned with spin systems on a regular lattice, with co-ordination number q, in the limit $q\to\infty$. This limit has been termed the high density limit by Brout [4] who first developed expansions for spin systems on lattices in inverse powers of the co-ordination number q, with the Curie-Weiss theory as leading term. More recently, Thompson [5] has proved that the $q\to\infty$ limit indeed results in the Curie-Weiss theory for Ising systems. However, the proof uses graph-theoretical methods and cannot be readily extended to other spin systems. Here, an algebraic method is developed that enables the treatment of both n-vector and quantum spin systems.

Consider N spins occupying the sites of a d-dimensional lattice specified by the d-tuples

$$\boldsymbol{i}=(i_1,i_2,\ldots,i_d)\in\mathbb{Z}^d \tag{1.2}$$

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