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The Dirac Operator with Highly Singular Potentials

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Abstract. Time-dependent scattering theory for a Dirac particle with highly singular potential is developed. Criteria for asymptotic completeness of wave operators are obtained, and an example is given of a potential which violates asymptotic completeness and the unitarity of the scattering operator. (Completeness breaks down for a regular sequence of values of the coupling constant.)

1. Introduction

This paper sets out to develop the scattering theory of a relativistic Dirac particle with potentials which may be highly singular. (For related work and references on the Dirac operator see [1—4].) The aim is to bring aspects of the theory relating to asymptotic completeness of wave operators and spectral properties of the Hamiltonian into line with results for the corresponding non-relativistic Schrödinger problem, and in particular with the results of [9—11]. (The *existence* of wave operators for highly singular potentials of short range is a consequence of the argument of Kupsch and Sandhas [5], cf. [1] for the necessary estimates.)

Section 2 deals briefly with the general theory of scattering by a potential which is singular on some closed, bounded set Σ of measure zero. As in the Schrödinger case, the Hilbert space decomposes into two orthogonal subspaces, consisting respectively of states in which, for large times, the particle with probability 1 approaches Σ, its kinetic energy becoming unbounded, and states in which the particle escapes to infinity. Apart from technical details (for example only a *local* condition need be imposed on the potential away from its singularities) this treatment follows closely that of [10]. For short range potentials, (strong) asymptotic completeness corresponds to the absence of states of the first kind.

We consider that the setting up of criteria for completeness logically precedes further consideration of the more unusual situation where completeness is violated. Section 3 proves completeness for a wide range of spherically symmetric short range potentials, singular or non-singular, and establishes for these potentials the absence of singular continuous spectrum. The main results, in Theorem 3, apply in