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## The Pole-Factorization Theorem in S-Matrix Theory\*

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**Abstract.** Previous derivations of physical-region discontinuity formulas in S-matrix theory make use of an ad hoc assumption according to which certain sets of singularities associated with mixed- $\alpha$  Landau diagrams cancel among themselves. The aim of the present work is to prove the simplest of these discontinuity formulas, namely, the pole-factorization theorem for a  $3\rightarrow 3$  equal-mass process below the 4-particle threshold, without using this mixed- $\alpha$  cancellation assumption. The result is derived from macro-causality, unitarity and two weak regularity assumptions on scattering functions and bubble diagram functions.

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## 1. Introduction

The basic quantities of interest in the study of systems of massive particles with short-range interactions are the scattering functionals  $S_{IJ}$  between sets I and J of

<sup>\*</sup> This work was supported by the United States Energy Research and Development Administration, and by Centre d'Études Nucleaires de Saclay