Commun. math. Phys. 56, 147—159 (1977)

Communications in
**Mathematical
Physics**

On the Uniqueness of the Invariant Equilibrium State and Surface Tension

C. Gruber and A. Hintermann*

Laboratoire de Physique Théorique, Ecole Polytechnique Fédérale de Lausanne, CH-Lausanne, Switzerland

A. Messager and S. Miracle-Sole

Centre de Physique Théorique, C.N.R.S., F-Marseille, France

Abstract. Symmetric Equilibrium States and their properties under duality transformation are investigated. Necessary and sufficient conditions are derived for equilibrium states to be transformed into equilibrium states by duality. It is shown that ferromagnetic systems satisfying those conditions have correlation functions bounded by those corresponding to the (+) and free boundary conditions. It is then proved than any Invariant Equilibrium State of a ferromagnetic system is transformed into an equilibrium state by duality and is thus unique if the states defined by the (+), and free boundary conditions coincide on the symmetric algebra. The existence of surface tension between two pure phases is established.

1. Introduction

In this paper, we investigate some consequences of the duality transformation[1] which are of interest for the study of ferromagnetic systems. It was recently established that, for all temperature, there exists a unique, symmetric, translation invariant, equilibrium state for the two-dimensional Ising model [2]; as was suggested in [3] the duality transformation was indeed a key to the proof of this result. The motivation of the following work relies upon the conjecture that for any lattice system, there should be a unique equilibrium state, which is invariant under the full symmetry group of the Hamiltonian, also called "*Invariant Equilibrium State*". We shall then derive general properties of symmetric equilibrium states and discuss certain consequences of the duality transformation.

In Section 3, we define symmetric equilibrium states by means of the solutions of equations which are well adapted for the study of duality. Necessary and sufficient conditions are then derived in Section 4 for a state to be transformed into an

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* Present address: Swiss Institute for Nuclear Research, CH-5234 Villigen, Switzerland

1 Introduced by H. A. Kramers and G. H. Wannier for the Ising model duality is a symmetry property inherent to lattice systems. See for instance [1] and references cited below