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Analyticity Properties and Many-Particle Structure in General Quantum Field Theory

III. Two-Particle Irreducibility in a Single Channel

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Abstract. In the framework of L.S.Z. field theory in the case of a single massive scalar field, the "two-particle irreducible" parts of the n-point functions (in any single channel and for arbitrary n) are defined as the solutions of a system of integral equations suggested by the perturbative framework. These solutions enjoy the analytic and algebraic properties of general n-point functions (up to possible polar singularities of generalized C.D.D. type). Morever it is shown that the completeness of asymptotic states in the two-particle spectral region is equivalent to the analyticity of the two-particle irreducible n-point functions in the corresponding regions of complex momentum space.

1. Introduction

The previous papers in this series [1, 2] were devoted to the first steps of the off-shell non-linear program of general quantum field theory, following the line of the many-particle structure analysis of Symanzik [3].

In this program an essential role is played by the (perturbative) notion of "p-particle irreducible (p.i.) part" of a Green's function (with respect to a certain channel), which has to be rigorously incorporated in the axiomatic framework.

The present paper is devoted to the study of this problem in the case $p=2$, namely to the extraction of two-particle singularities from the n-point functions of a local field.

In other words[1], for any partition $(I, N\backslash I)$ of the set of indices $N=\{1,2,\dots,n\}$, $n\geqq 2$ arbitrary, we want to define a function $G^{I,N\backslash I}$ enjoying the following properties:

a) $G^{I,N\backslash I}$ is a general n-point function [1], i.e. it is analytic in the n-point primitive domain $D^{(n)}$ and its real boundary values satisfy Steinmann relations[2].

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[1] The notations are those of [2]. For simplicity, we restrict to the case of a single mass m in the spectrum

[2] For original works concerning the primitive structure of n-point functions, see [4–7]