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Charged Particles in External Fields

II. The Quantized Dirac and Klein-Gordon Theories

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Abstract. The second-quantized Dirac and Klein-Gordon equations with external fields are solved. It is shown that the interpolating field is local and satisfies the Yang-Feldman equations. The Capri-Wightman approach and the Friedrichs-Segal approach are shown to lead to the same unitary S-operator. The evolution operator and S-operator are studied. A divergence-free perturbation expansion of the S-operator is derived and the connection with the Feynman-Dyson series is established.

1. Introduction

In a previous paper [1] (which we shall refer to as I) a number of perturbation-theoretic results were obtained which were applied to the classical Dirac and Klein-Gordon equations with external fields. The quantized theories will now be considered.

As detailed in [2] (referred to as B), one can treat the external field problem in a rigorous way by using the operators from the classical theory to generate transformations of the field operators on Fock space which amount to Bogoliubov transformations. If such a transformation is implementable the resulting Fock space operator is regarded as the physical operator corresponding to the unphysical operator on the classical Hilbert space. This approach, which goes back to Friedrichs [3], was further developed in [4–8, 2]. A closely related but more algebraic approach, inspired by the ideas of Segal [9], was used in [10–17].

A rather different strategy, based on the Yang-Feldman equations, was initiated by Capri [18] and further developed by Wightman [19]; it can be used for any generalized Dirac equation. Yet another treatment, using ideas from renormalization theory, was recently given by Bellissard [20, 21].

On a formal level the scattering of (especially spin-$\frac{1}{2}$) particles at external fields has been considered some time ago. Some references are [22–24]. Detailed accounts of the formal theory can be found in the books by Schweber [25] and Thirring [26].

One of the main results of this paper is that for (massive, relativistic, charged) spin-0 and spin-$\frac{1}{2}$ particles in external fields which are test functions on space-time