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# First and Second Quantised Neutron Diffusion Equations

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**Abstract.** We show that a quantised linear Boltzmann equation can be obtained as an exact contracted form of a second quantised neutron diffusion equation in the weak coupling limit.

## § 1. Introduction

Let $\mathscr{F}$ be the fermion Fock space built from a single particle space $\mathscr{H}=L^2(\mathbb{R}^3)$ and consider the evolution equation

$$\varrho'(t)=-i[H,\varrho]-\lambda(R\varrho+\varrho R)/2+\lambda J(\varrho)\,, \tag{1.1}$$

where $\varrho(t)$ lies in the space $V=\mathscr{T}_s(\mathscr{F})$ of self-adjoint trace-class operators on $\mathscr{F}$. Here $H$ is the free Hamiltonian on $\mathscr{F}$ which equals $-\Delta$ on $\mathscr{H}$. The unbounded positive linear map $J:V\to V$ is defined by

$$J(\varrho)=\int_{\mathbb{R}^3} B_x\varrho B_x^* dx \tag{1.2}$$

where

$$B_x=a^*(f_x^3)a^*(f_x^2)a(f_x^1)\,. \tag{1.3}$$

In Equation (1.3), $a$ and $a^*$ are the fermion field operators smeared by the test functions $f_x^i$ in Schwartz space $\mathscr{S}$. Moreover $f_x^i$ is defined as the translate by a distance $x\in\mathbb{R}^3$ of $f^i$ and it is supposed that $f^2$ and $f^3$ have disjoint supports in momentum space. Finally $\lambda>0$ and

$$R=\int_{\mathbb{R}^3} B_x^*B_x dx\,. \tag{1.4}$$

Without further loss of generality we suppose that $f^2$ and $f^3$ have unit norm in $\mathscr{H}$.

The above phenomenological evolution equation describes a variable number of neutrons moving in a translation invariant external reservoir of unstable atoms, which they can induce to decay emitting further neutrons. It is shown in [2] that Equation (1.1) has the solution

$$\varrho(t)=T_t\{\varrho(0)\} \tag{1.5}$$