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A Restriction on the Topology of Cauchy Surfaces in General Relativity

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Abstract. It is shown that, in a globally hyperbolic and geodesically complete space-time, a part of a partial Cauchy surface that is bounded by a uniformly convex sphere is compact and simply connected.

Introduction

There are several indications that we might reasonably disregard the possibility that the universe has anything other[1] than a very simple topological structure. Geroch [1] has shown that an asymptotically simple and empty space-time is homeomorphic to $\mathbb{R}^4$. A result by Hawking[2] on future asymptotically predictable space-times would seem to limit the possible topology of those space-times. There have also been some results on the possibility of the topology of space changing with time [2, 3, 4].

It would be difficult to prove anything about the topology of space near to singularities, due to the arbitrariness associated with them. As the universe is expected to be globally past incomplete it would also be difficult to say anything about its topology as a whole without some uniformity principle. For these reasons we limit ourselves to a part of space that can be "enclosed by" a well-behaved sphere. There is no restriction on the size of the sphere. We show that geodesic incompleteness is associated with topological peculiarities in such a part of space. That such incompleteness should occur separate from either collapsed objects or the beginning or end of the universe is objectionable.

The theorems are only proved for space-times with global Cauchy surfaces. There seems to be little reason to suppose that firstly the universe has one[3], and secondly the results cannot be proved without one. It is a serious disadvantage, however, as the results are intended to be applicable to non-global topology without reference to, possibly unrealistic, global properties.

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1 E. g., multiple connectedness

2 Proposition 9.2.1 of Reference [5]

3 See, for example, Reference [5]