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Spectra of Liouville Operators

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Abstract. Spectra of the generators of time translations ("Liouville operators") on representation spaces determined by thermodynamic equilibrium states are compared and their nature is investigated.

1. Introduction

For macroscopic systems the density of energy levels is approximately $(\Delta E)^N$ where ΔE is the energy above the groundstate and N the number of degrees of freedom. Because of this enormous growth of level density in the thermodynamic limit one often says that the energy spectrum becomes continuous in that limit.

It is the purpose of this paper to study spectral properties of relevant objects, that govern the dynamics of quantum systems. In the quantum theory of a finite number of particles the above mentioned questions are discussed in terms of the spectral properties of the Hamiltonian, i.e. the generator of time-translations, of the system. In a quantum mechanical treatment of a thermodynamic system, i.e. of a system consisting of an infinite number of particles in infinite space with a finite density, the generator of time-translations is not unambiguously defined, let alone its spectrum.

We shall assume that we have a C^*-algebra $\mathfrak{A}$ of quasi-local observables with local algebras isomorphic to $\mathscr{B}(\mathfrak{h}_V)$, i.e. the local algebras consist of all bounded operators on the Hilbert-space $\mathfrak{h}_V$ that is pertinent to the description of a quantum system inside a volume V. The dynamics is assumed to be given by a one-parameter group of automorphisms α_t of $\mathfrak{A}$ that admits of a K.M.S.-state and satisfies some regularity conditions to be specified in section 2b. As we shall see in section 2b these regularity conditions permit us to construct a separable C^*-algebra $\mathfrak{A}_0$ inside $\mathfrak{A}$, that is $\sigma(\mathfrak{A}, N)$ dense in $\mathfrak{A}$. (Here N is the set of locally normal states on $\mathfrak{A}$ and the $\sigma(\mathfrak{A}, N)$ topology on $\mathfrak{A}$ is the weak topology defined by N on $\mathfrak{A}$.) The construction of $\mathfrak{A}_0$ depends on α_t and is such that α_t acts strongly continuous on $\mathfrak{A}_0$, i.e. $\|\alpha_t(A)-A\| \xrightarrow[t\to 0]{} 0$ for $A \in \mathfrak{A}_0$.