Groups of Automorphisms with Spectrum Condition and the Lifting Problem

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Abstract. We show that a continuous one-parameter group $\{\alpha_t\}$ of auto-morphisms of a separable C^*-algebra A satisfies a spectrum condition if and only if it is the limit, pointwise on A, uniformly on compact subsets of $\mathbb{R}$ of a sequence of inner automorphism groups whose positive generators do not increase too fast. Moreover we prove in this case that if π is a surjective morphism of a separable C^*-algebra B on A then there is a similar group $\{\bar{\alpha}_t\}$ of automorphisms of B such that $\pi \circ \bar{\alpha}_t = \alpha_t \circ \pi$ for all t.

1. Introduction

Most of the current work in operator algebra is influenced by problems arising in mathematical physics and is concerned with situations involving a C^*-algebra A (the observables) and a continuous one-parameter group $\{\alpha_t\}$ of automorphisms of A (the time development). We can always represent A as operators on some Hilbert space H such that $\alpha_t(x) = u_t x u_t^*$ for all x in A and t in $\mathbb{R}$, where $u_t = \exp(ith)$ and h is a self-adjoint operator (the hamiltonian) in H. The case where h is (or can be chosen to be) semi-bounded is interesting for the applications (corresponding to positive energy) and quite attractive from a mathematical point of view. We say in this situation that $\{\alpha_t\}$ satisfies a spectrum condition in the given representation of A.

In this paper we deal with the most restrictive version of the above, namely that $\{\alpha_t\}$ satisfies a spectrum condition in the universal representation of A. Unfortunately, this assumption is not tenable in more realistic models, so our results must be taken more as a guideline what to expect, than as a valid descrip-tion of a physical situation. We show that there is an increasing (but unbounded) sequence $\{h_n\}$ in A_+ which converges (in a sense to be made precise) such that the sequence $\{\exp(ith_n) \cdot \exp(-ith_n)\}$ of inner automorphism groups converges to $\{\alpha_t\}$ pointwise on A and uniformly on compact subsets of $\mathbb{R}$.

The paper originated in an effort to extend the lifting theorem for auto-morphism groups to groups satisfying a spectrum condition. It was shown in