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Sommerfeld-Watson Representation for Double-Spectral Functions

III. Crossing Symmetric Pion-Pion Scattering Amplitude with Regge Poles

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Abstract. We demonstrate the existence of a solution of the nonlinear pion-pion equations that incorporate crossing symmetry, unitarity and l-plane meromorphy. In particular, we show how to guarantee the boundedness of the partial waves as $s\to\infty$, even when some Regge trajectories rise beyond unity.

1. Introduction

In previous papers of this series [1, 2], nonlinear equations for the partial-wave amplitudes were set up to guarantee unitarity and analyticity in the right half of the angular momentum plane. The treatment in Ref. [1] contained only a direct channel, as in potential theory; but that of Ref. [2] allowed for full three-channel crossing symmetry. The potential case was considered also in a later paper [3], in which improved bounds for the Legendre functions allowed contact to be made with ordinary energy-independent potential scattering. In all this work, no singularities were allowed in the right half of the angular momentum plane.

The present work is devoted to setting up the corresponding Sommerfeld-Watson equations when there are Regge poles for $\mathrm{Re}\, l>0$ [4]. For simplicity in notation, we write down only one Regge pole; but there is no difficulty in handling any finite number of them. The equations of the present paper should be regarded as a representation for a valid crossing-symmetric Regge amplitude, rather than as a dynamical scheme for its calculation. Thus, given the Regge pole position, $\alpha(s)$, and residue, $\beta(s)$, and the central inelastic double spectral function, $v(s,t)$, we write down equations that the amplitude must satisfy. No attempt is made to calculate α and β from v, in the way that the Regge parameters are calculable in a potential theory. In a relativistic theory, where one does not have the constraint implied by the existence of a given potential, one would not expect $\alpha(s)$ and $\beta(s)$ to be determined uniquely by $v(s,t)$. Nevertheless, elastic unitarity may serve to restrict the allowed functions, $\alpha(s)$ and $\beta(s)$, once $v(s,t)$ is given. Our treatment of the equations as a nonlinear mapping indicates that this may be true, since it turns out that unitarity is not satisfied by fixed points of the system,