Commun. math. Phys. 51, 55—66 (1976)

Communications in
Mathematical
Physics

Quasi-free States and Automorphisms of the CCR-Algebra

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Abstract. We show that any automorphism of the CCR algebra, leaving the quasi-free states globally invariant, is monoparticular.

1. Introduction

The analogous problem for infinite Fermi systems has been studied in two recent papers. In [2] Hugenholtz and Kadison assume that the gauge invariant quasi-free states are globally invariant under the action of an automorphism whereas in [6] Wolfe treats also the situation where all quasi-free states are globally invariant. The conclusion that the automorphism is monoparticular is reached by completely different methods.

The method used in this paper seems again to be quite different from those used in [2] and [6]. The main idea is to introduce an order relation in the set of quasi-free states. We say that $\omega_1 \lesssim \omega_2$ if $\omega_1 \leqq \gamma\omega_2$ for some $\gamma \in \mathbb{R}$; $\lesssim$ defines an ordering because of the exponential character of the quasi-free states.

The main use of $\lesssim$ is to show that adding scalars to the fields $a_\omega(\cdot)$ and $a^*_\omega(\cdot)$ in the representation of a given quasi-free state ω corresponds to the same kind of transformation for the fields $a_{\omega\circ\alpha}(\cdot)$ and $a^*_{\omega\circ\alpha}(\cdot)$, where α denotes the automorphism in question.

2. Preliminaries [3–5]

Let $\mathscr{H}$ be a separable (possibly finite dimensional) Hilbert space over $\mathbb{C}$ with inner product $(\cdot\,|\,\cdot)$ (antilinear in the first component) and H its underlying real Hilbert space. $H=\mathscr{H}$ as a set and the inner product $\langle\cdot\,|\,\cdot\rangle$ of H is given by

$$\langle\phi|\psi\rangle = \mathrm{Re}(\phi|\psi) \qquad \phi,\psi\in H\,.$$

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