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On the Bound State in Weakly Coupled $\lambda(\varphi^6 - \varphi^4)_2$

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Abstract. We consider the $\lambda(\varphi^6 - \varphi^4)$ quantum field theory in two space-time dimensions. Using the Bethe-Salpeter equation, we show that there is a unique two particle bound state if the coupling constant $\lambda > 0$ is sufficiently small. If m is the mass of single particles then the bound state mass is given by

$$\varkappa_B(\lambda) = 2m\left(1 - \frac{9}{8}\left(\frac{\lambda}{m^2}\right)^2 + \mathcal{O}(\lambda^3)\right).$$

1. The Bound State Problem

We consider relativistic scalar boson quantum field theories in two dimensional space-time with polynomial interactions and we discuss some properties of bound states below the two particle threshold. For the model with interaction polynomial $P(\varphi) = \lambda(\varphi^6 - \varphi^4)$, coupling constant $\lambda > 0$ and bare mass m_0, bound states are known to exist if λ/m_0^2 is sufficiently small. This result is implicit in the combination of the two papers [4] and [7]. In the first paper, Glimm et al. argue that the $\lambda(\varphi^6 - \varphi^4)$ model has mass spectrum above the one particle mass shell and below the two particle threshold. (They assumed that the physical mass $m = m(\lambda, m_0)$ has an asymptotic expansion as a function of λ near $\lambda = 0$; this was subsequently proved in [2].) Secondly, Spencer and Zirilli, based on estimates by Spencer [6], showed that for any even P the mass operator has only discrete spectrum below $2m$, and that on each eigenspace of the mass operator the representation of the Poincaré group decomposes into a finite sum of irreducible representations. Thus the spectrum in question is interpreted as bound state particles.

In this paper we continue the study of the $\lambda(\varphi^6 - \varphi^4)$ model and sharpen the above results. It is convenient (though not essential) to choose the bare mass $m_0 = m_0(\lambda)$ such that the physical mass $m = m(\lambda, m_0(\lambda))$ is fixed [2]. Our main result is:

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