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Operator Product Expansions on the Vacuum in Conformal Quantum Field Theory in Two Spacetime Dimensions*

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Abstract. Let $\varphi_1(x)$ and $\varphi_2(y)$ be two local fields in a conformal quantum field theory (CQFT) in two dimensional spacetime. It is then shown that the vector-valued distribution $\varphi_1(x)\varphi_2(y)|0\rangle$ is a boundary value of a vectorvalued holomorphic function which is defined on a large conformally invariant domain. By group theoretical arguments alone it is proved that $\varphi_1(x)\varphi_2(y)|0\rangle$ can be expanded into conformal partial waves. These have all the properties of a global version of Wilson's operator product expansions when applied to the vacuum state $|0\rangle$. Finally, the corresponding calculations are carried out more explicitly in the Thirring model. Here, a complete set of local conformally covariant fields is found, which is closed under vacuum expansion of any two it its elements (a vacuum expansion is an operator product expansion applied to the vacuum).

I. Introduction

Some time ago partial wave expansions of the euclidean Greensfunctions (i.e. the Schwingerfunctions) of a CQFT have been established [1]. These expansions are useful to solve the nonlinear dynamical integralequations and also help to study the implications of locality. However, when one tries to express Osterwalder-Schrader-positivity (i.e. the euclidean counterpart of ordinary Wightman-positivity) in terms of the conformal partial waves, a complicated process of analytic continuation in the expansion parameters is needed [2]. In fact, one performs something like an inverse Sommerfeld-Watson-transform. The resulting discrete expansion is then termwise positive. Moreover the series looks exactly like a globally valid form of an operator product expansion applied to the vacuum. The above mentioned manipulations with the euclidean partial waves can only be done under suitable technical assumptions. For instance, to prove the validity of the inverse Sommerfeld-Walson-transform one must make sure that the partial waves have appropriate asymptotic properties in the expansion parameters.

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