Remark on the Energy Spectrum of a Decaying System

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Abstract. Some time-evolution operators of a general unstable system lead
to unphysical spectrum (unbounded below) of the total Hamiltonian. Various
necessary conditions for boundeness of the spectrum are known. It is shown
here, how this spectrum can be determined, which, in particular, gives the
sufficient condition.

1. Introduction

It is well-known, that the exponential decay law, though is confirmed
experimentally in a wide range, has an unphysical property; it corresponds to the
energy spectrum without (lower) bound. Also some similarly behaving decay
laws in the general scheme for description of unstable systems (developed mostly
in the last few years – see e.g. Refs. [1–5]) exhibit this unpleasant feature. Various
authors [3–7] deduce different conditions for the "reduced evolution operator"
(time-evolution operator of the unstable system itself), under which spectrum
of the total Hamiltonian is (below) unbounded. Nevertheless, the same question
arises for those reduced evolution operators which do not obey any of the men-
tioned conditions. We shall show here how the spectrum can be determined to a
given reduced evolution operator. It gives, in particular, a possibility to decide
whether a reduced evolution operator corresponds to a total Hamiltonian with
a spectrum bounded below.

In Section 2 we collect the assumptions in our description of unstable systems
and introduce some notions. Section 3 is devoted to derivation of relations
between a reduced evolution operator and spectrum of the corresponding total
Hamiltonian. A simple criterion is given for the case when the state Hilbert space
of an unstable system is finite-dimensional. The last section contains discussion
of the results, a special attention being payed to connections of decay laws to the
total Hamiltonian.