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Scattering States and Bound States in $\lambda\mathscr{P}(\phi)_2$*

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Abstract. By analyzing the Bethe-Salpeter equation for even $\lambda\mathscr{P}(\phi)_2$ models we show that for weak coupling the mass spectrum is discrete and of finite multiplicity below $2m$. Moreover on even states of energy less than $4(m-\varepsilon)$ we show that the S matrix is unitary. Here m is the physical mass and $\varepsilon=\varepsilon(\lambda)\to 0$ as $\lambda\to 0$. Our results rely essentially only on a simple assumption about the analyticity of the Bethe-Salpeter kernel which has been verified for weak coupling. For the interaction $\lambda\phi^4$, $(\lambda/m_0^2\ll 1)$ we show that there are no even bound states of energy less than $4(m-\varepsilon)$.

Introduction

We investigate the energy-momentum spectrum for even $\lambda\mathscr{P}(\phi)_2$ models via the Euclidean Bethe-Salpeter equation. Let $P=(P^0,P^1)$ be the energy-momentum operator acting on the Hilbert space of states $\mathscr{H}$ and define $\Omega\in\mathscr{H}$ to be the vacuum. The first results concerning the spectrum of P were established by Glimm et al. [1, 2]. By using a weak coupling cluster expansion, they showed that the closure of the span of

$$\Omega,\, e^{ix^0P^0}\phi_0(f_1)\Omega,\, \dots,\, e^{ix^0P^0}\prod_i^N \phi_0(f_i)\Omega,\, f_i\in C_0^\infty(\mathbb{R})$$

contains all states of energy less than $(N+1)(m-\varepsilon)$ for λ (depending on N) sufficiently small. Here $\varepsilon(\lambda)\to 0$ as $\lambda\to 0$ and $\phi_0(f_i)$ denotes the time zero field smeared with f_i. It was also shown that for even $\mathscr{P}$ the mass operator restricted to the odd subspace of $\mathscr{H}$ has exactly one eigenvalue m on the interval $[0,3(m-\varepsilon)]$. As a result the Haag-Ruelle theory [3] yields the existence of an isometric S matrix. It has recently been shown that $S\neq 1$ and is asymptotic in λ [4, 5, 13]. For the special case of $\lambda\phi^4$, bound states of energy less than $2m$ were excluded by using correlation inequalities [2].

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