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Stability and Equilibrium States of Infinite Classical Systems

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Abstract. We prove that any stationary state describing an infinite classical system which is "stable" under local perturbations (and possesses some strong time clustering properties) must satisfy the "classical" KMS condition. (This in turn implies, quite generally, that it is a Gibbs state.) Similar results have been proven previously for quantum systems by Haag et al. and for finite classical systems by Lebowitz et al.

1. Introduction

It is generally accepted that the appropriate microscopic description of both equilibrium and nonequilibrium properties of bulk matter is via "macroscopic states". These states are (when quantum effects are unimportant) probability measures on the phase space K of an infinite system of indistinguishable particles moving in $\mathbb{R}^3$ [1, 2]. When the system is in equilibrium it is assumed that the appropriate macroscopic state is a Gibbs (or equilibrium) measure at some temperature β^{-1} and fugacity z. There are several alternative ways of describing these measures, e.g. the DLR equations, the Kirkwood-Salsburg equations, etc. These are however all essentially equivalent [3, 4]: the infinite volume Gibbs states corresponding (as they presumably should physically) to an appropriate limit of finite volume grand canonical ensembles. Other finite volume ensembles such as the microcanonical and the canonical are also expected to have Gibbs states as limits [5].

In this paper we investigate the problem of justifying the use of Gibbs measures for infinite classical systems. (The analogous problem for finite classical system was treated in [6].) Of course, any measure ω which describes the state of a system in equilibrium must be stationary, i.e. invariant under the time evolu-

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