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Some Remarks on Fröhlich's Condition in $P(\phi)_2$ Euclidean Field Theory

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Abstract. We derive Fröhlich's condition as the KMS condition on a suitable algebra and time translation. Next we consider Fröhlich's condition and its variance and prove their equivalence in a general setting. Finally we mention some results which follow from the latter condition.

§ 1. Introduction

In the $P(\phi)_2$ Euclidean field theory there are some "equilibrium equations" of probability measures on (Q, Σ) where Q may be realized as $\mathscr{D}' = \mathscr{D}'_{\text{real}}(R^2)$ or $\mathscr{S}' = \mathscr{S}'_{\text{real}}(R^2)$ and Σ as the σ-algebra generated by cylinder sets. They are expected to completely characterize infinite volume theories with given interaction and bare mass. One of them is given by Guerra, Rosen and Simon [5], which is a version of the DLR equations in classical statistical mechanics. Recently Fröhlich [3] observed that such measures are quasi-invariant and obtained another characterization which is expressed by Radon-Nikodym derivatives: Let ν be a probability measure and satisfy

$$\frac{d\nu(\phi+g)}{d\nu(\phi)} = \exp\{-\phi(\mu^2 g) - \tfrac{1}{2}\|\mu g\|_{L^2}^2 - [\mathscr{U}(\phi+g) - \mathscr{U}(\phi)]\} \tag{1.1}$$

for any g in $\mathscr{D}(R^2)$ where μ^2 denotes $-\Delta + m_0^2$ and

$$[\mathscr{U}(\phi+g) - \mathscr{U}(\phi)] = \int \{:P(\phi(x)+g(x)): - :P(\phi(x)):\} d^2x \tag{1.2}$$

for an interaction polynomial P bounded below and a bare mass m_0. In this case ν should be called an "equilibrium measure" if it further satisfies the physical conditions (e.g. Osterwalder-Schrader-Nakano positivity). Fröhlich showed that the two characterizations are equivalent.

In the present note our first purpose is to show the equivalence of the condition (1.1) with the KMS condition relative to suitable time translation automorphisms as Brascamp [2] did in the case of classical lattice gas (see Theorem 3). In this way the similarity of the $P(\phi)_2$ Euclidean theory with classical statistical mechanics becomes more complete.