Commun. math. Phys. 47, 75—79 (1976)

Communications in
Mathematical
Physics

A Conventional Proof of Kerr's Theorem*

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Abstract. A proof of Kerr's Theorem for generating geodesic and shearfree null congruences in Minkowski space is given in the Newman-Penrose null tetrad formalism.

Kerr's Theorem gives an algorithm for obtaining the most general geodesic and shearfree null congruence (GSF congruence) in Minkowski space. The theorem is important for constructing the Kerr-Schild spacetimes [1] and is also essential in the twistor treatment of zero rest mass free fields in Minkowski space [2]. The published proofs of Kerr's Theorem must be either extracted indirectly from the Kerr-Schild field equations [1], or they must be translated from twistor language which is unfamiliar to many relativists [2]. Nowhere, to our knowledge, is there to be found a straightforward proof of the theorem in familiar terms. The purpose of this note is to remedy this situation by presenting a proof of Kerr's Theorem in the Newman-Penrose null tetrad formalism [3].

Let the metric of Minkowski space be written as

$$ds^2 = du dv - d\zeta d\bar{\zeta} = (l_a n_b + n_a l_b - m_a \bar{m}_b - \bar{m}_a m_b) dx^a dx^b \,, \tag{1}$$

where l_a, n_a, m_a, $\bar{m}_a$ is a *constant* normed null tetrad. Then Kerr's Theorem states that the most general analytic GSF null congruence ξ_a is given by either $\xi_a = n_a$ or by

$$\xi_a = l_a + Y\bar{m}_a + \bar{Y} m_a + Y\bar{Y} n_a \,, \tag{2}$$

where $\bar{Y}$ is a complex function of the coordinates u, v, ζ, $\bar{\zeta}$ defined implicitly by $F=0$, where

$$F = F(\bar{Y}, u + \bar{Y}\zeta, \bar{\zeta} + \bar{Y}v)$$

is an arbitrary complex analytic function of its three arguments. [The case $\xi_a = n_a$ can be thought of as arising from the form (2) in the limit $Y \to \infty$.]

* Supported in part by NSF Grant GP-43844-X.

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