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Convergence Theorems for Renormalized Feynman Integrals with Zero-Mass Propagators

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Abstract. A general momentum-space subtraction procedure is proposed for the removal of both ultraviolet and infrared divergences of Feynman integrals. Convergence theorems are proved which allow one to define time-ordered Green functions, as tempered distributions, for a wide class of theories with zero-mass propagators.

I. Introduction

Until recently, the applicability of Zimmermann's momentum-space subtraction procedure [1, 2] for Feynman integrals was limited to cases with no vanishing masses. To be sure, zero-mass theories could be discussed, but only as limits of corresponding massive theories, and even then only the limits for non-exceptional Euclidean momenta could be taken legitimately. The aim of the present work is to extend the convergence theorems of Zimmermann to a very general class of momentum-space Feynman integrals with arbitrary non-negative mass parameters.

The subtraction scheme adopted below evolved from one used by the author and Zimmermann [3] to formulate the massless A^4 model (see also [4]). The essential idea is the following: all subtractions are made at vanishing momentum, but some of them, including the ones otherwise expected to give rise to infrared divergences (mass singularities) by naive power counting, are made at a non-zero value of the mass. The absolute convergence of the subtracted Feynman integrals in the massless A^4 model has been proved [5] for non-exceptional (in the Euclidean sense) external momenta and non-zero ε in the denominator factors. Similar treatments have been given for the Goldstone model and Abelian Higgs model [3, 6]. Unfortunately the methods of these references are not sufficient to treat models with renormalization parts with three external boson lines (e.g. massless scalar QED, non-Abelian Yang-Mills models). If one gives the three-

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