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Dissipations and Derivations

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Abstract. We show a usefulness of the notion of "dissipative operators" in the study of derivations of C^*-algebras and prove that the closure of a normal $*$-derivation of UHF algebra satisfying a special condition is a generator of a one-parameter group of $*$-automorphisms.

§ 1. Introduction

Recently various authors have studied unbounded derivations of C^*-algebras [2–4, 6, 7, 10, 11, 13]. In particular Powers and Sakai [10] have studied unbounded derivations of UHF algebra.

The purpose of the present note is to show a usefulness of the notion of "dissipative operators" [9, 17] in the study of derivations of C^*-algebras.

Our first result is that an everywhere defined "dissipation" is bounded, which implies the well-known theorem concerning derivations [5, 12].

Our second result is about a normal $*$-derivation of UHF algebra satisfying a special condition discussed in [1, 10, 14, 15]. For such a $*$-derivation, we prove that its closure is a generator of a one-parameter group of $*$-automorphisms. As its application we consider one-dimensional lattice system.

§ 2. Bounded Derivation

Let $\mathfrak{A}$ be a Banach space. For each $x \in \mathfrak{A}$ there is at least one non-zero element f of the dual Banach space $\mathfrak{A}^*$ such that $\langle x, f \rangle = \|x\| \cdot \|f\|$ by the Hahn-Banach theorem. An f_x denotes one of them throughout this note.

Definition 1. [9] A linear map γ with domain $\mathscr{D}(\gamma)$ in a Banach space is called dissipative if there is an f_x such that

$$\mathrm{Re}\langle \gamma x, f_x \rangle \leqq 0$$

for each $x \in \mathscr{D}(\gamma)$.

Definition 2. A linear map δ with domain $\mathscr{D}(\delta)$ in a Banach space is called derivative if there is an f_x such that

$$\mathrm{Re}\langle \delta x, f_x \rangle = 0$$

for each $x \in \mathscr{D}(\delta)$.

Let $\mathfrak{A}$ be a C^*-algebra. A linear map δ of $\mathfrak{A}$ is called a derivation if it satisfies

$$\delta(xy) = \delta(x)y + x\delta(y)$$