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Zeeman Topologies on Space-Times of General Relativity Theory

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Abstract. In 1964 Zeeman published a paper showing [independently of Alexandrov (1953)] that the causal structure of the light cones on Minkowski space M determines the linear structure of M. This initiated the question whether a topology (more physically than the ordinary one) on M, related to the light cones also implies the linear structure of M. In 1967 Zeeman defined such a new topology – here called Zeeman-topology $\mathfrak{Z}_0$ – on Minkowski space and solved this question for M. In that paper he asked whether it is possible to generalize this program to general relativity. Two of his main questions were: (a) What is the structure of the group $G(S)$ of all homeomorphisms of a space-time S with respect to the general relativistic analogue of $\mathfrak{Z}_0$ (defined in § 3)? (b) What are the world lines (defined in § 1) with respect to $\mathfrak{Z}_0$? Without any restrictions on the space-time S we will give the answers: (a) $G(S)$ is the group of all homothetic transformations on S (for an explicite discussion of this result we refer to § 5). (b) World lines are broken geodesics. Including external fields (like Maxwell fields and deviations of $\mathfrak{Z}_0$) the answer (b) can be generalized in different physical directions; cf. § 3.

Ein Fernrohr wird gezeigt, womit man seinen eigenen Rücken sieht.
Es führt durchs Weltall deinen Blick im Kreis zurück auf dein Genick.
Zwar braucht es so geraume Frist, daß du schon längst verstorben bist,
doch wird ein Standbild dir geweiht, empfängt es ihn zu seiner Zeit.

Christian Morgenstern, „Böhmischer Jahrmarkt"

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