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On Invariant and Covariant Schwartz Distributions in the Case of a Compact Linear Group

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Abstract. A proof is given for the representations of invariant and covariant (Schwartz) distributions on R^n, which are often used in theoretical physics. We express invariant distributions as distributions of standard polynomial invariants and decompose covariant distributions in standard polynomial covariants. Our consideration is restricted to compact groups acting linearly on R^n. The representation for invariant distributions is obtained provided the standard invariants form an algebraically independent generating set in the ring of invariant polynomials. As for the standard covariants we assume that in the class of covariant polynomials they provide a unique decomposition into a sum of the standard covariants multiplied with invariant polynomials.

Introduction

It is part of the folklore of mathematical physics that distributions on the Euclidean space R^n, invariant with respect to a classical group (acting linearly on R^n), can be represented as "distributions" of a fixed (finite) family of standard polynomial invariants (provided that the invariants separate orbits, at least those of some kind of "regularity"). Similarly, it is often believed that covariant distributions can be decomposed into a sum of a fixed (finite) family of standard polynomial covariants multiplied with invariant distributions.

Special results in this direction are provided by the descriptions of rotation invariant distributions of one [1] or two [2] vectors and Lorentz invariant distributions of one vector [3]. (Concerning Lorentz covariant distributions of one vector see e.g. [4], Section 3.) Prior to such a description, one needs a choice of standard polynomial invariants (or covariants) which yield resolution of the corresponding algebraic problem on invariant (or covariant) polynomials. Of interest are more general situations which exhibit a close relationship between the distribution theoretic and the algebraic problem.

The purpose of this paper is to show that in the case of a compact linear group the desired representation for invariant (resp. covariant) distributions on R^n exists and is unique (in a certain sense), provided that the corresponding representation in the class of polynomials exists and is unique with respect to a given family of standard polynomial invariants (resp. covariants). By this means our