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Integrable Hamiltonian Systems and Interactions through Quadratic Constraints

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Abstract. O_n-invariant classical relativistic field theories in one time and one space dimension with interactions that are entirely due to quadratic constraints are shown to be closely related to integrable Hamiltonian systems.

I. Introduction

Even in one space dimension, relativistically invariant classical field theories defining integrable Hamiltonian systems with a non-trivial, momentum dependent scattering matrix, are not in oversupply. Actually, up to equivalence and slight modifications there is only one such model available, the celebrated sine-Gordon equation [1–3].

In this paper we shall present a whole series of non-equivalent relativistically invariant field theories in one time and one space dimension, each having a one parameter family of Backlund transformations and an infinite number of known integrals of motion. These conserved quantities are associated with covariant local conserved currents for which the family of Backlund transformations serves as a generating functional. Further, each one of these models has non-trivial momentum dependent scattering, and possesses stationary finite energy solutions: the solitons of the sine-Gordon theory.

By a procedure explained below ("reduction"), the series of new models is obtained from O_n-invariant Lagrangian field theories whose interaction arises solely from the condition that the values of the field functions be constrained to the surface of a sphere (describing a homogeneous space for O_n). The new examples should be viewed as generalizations (involving more and more fields) of the sine-Gordon theory, which corresponds to the chiral symmetry group O_3 (To O_2 there corresponds the theory of a free massless field). The connection with the O_n-invariant chiral theories allows for a simple geometrical interpretation of various computational manipulations in the new models. For $n \leq 6$ we set up the linear eigenvalue equation (for the characteristic initial value problem), which is the key to the inverse scattering method [4, 5]. We determine the evolution of the spectral data and thereby solve the characteristic initial value problem.