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Infrared Convergence of Feynman Integrals for the Massless A^4-Model

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Abstract. For the massless A^4-model it is proved that renormalization can be formulated such that each Feynman diagram yields an ultraviolet and infrared convergent contribution to the Green's functions.

1. Introduction

In a previous paper a new renormalization scheme was proposed for theories with zero-mass propagators. The characteristic feature of this method is that subtraction terms involve massive denominators so that no new infrared infinities are introduced by making subtractions at zero external momenta. So far the method has been applied to the massive A^4-model, the Goldstone and the pre-Higgs model in Ref. [1], as well as the Higgs model by Clark [2]. Presently under consideration is the application [3, 4] to the pure Yang-Mills field as an extension of the work by Becchi, Rouet, and Stora [5] on non-Abelian gauge theories. For all models considered the new subtraction scheme yields ultraviolet and infrared convergent contributions for each Feynman diagram separately. This eliminates the need of discussing cancellations of infrared infinities by cumbersome limiting procedures.

The purpose of this paper is to present a complete and rigorous convergence proof for the massless A^4-model as an application of a general power counting theorem [6]. The extension to the other models treated in Refs. [1] and [2] is straightforward.

After some remarks on the general form of the renormalized integrands (Section 1) the convergence of Feynman integrals is proved for all diagrams which do not contain internal self-energy insertions. In Sections 3 and 4 the general case is reduced to the task of verifying dimensional rules for certain expressions involving massless propagators only. These rules are checked recursively in Section 5 and 6 using the method of propagator product expansions.

2. General Properties of the Renormalized Integral

For the definition of the renormalized integrand R_Γ of a Feynman diagram Γ we refer to Section IIB of Ref. [1]. We further define

$$\bar{R}_{\varrho s^\varrho} = S_\varrho \sum_{U \in \mathscr{F}'_\varrho} \sum_{\gamma \in U} (-\tau_\gamma S_\gamma) I_\varrho(U) \tag{2.1}$$