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Ferromagnetic Spin Systems at Low Temperatures

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Abstract. Finite-spin systems with ferromagnetic, finite range interactions are considered. Using Ruelle's theorem on zeros of polynomials contracted according to Asano, analyticity of pressure and correlation functions is proved. A description of all translation invariant equilibrium states at low temperatures for a large class of systems is given.

Introduction

We develop here further the technique of [13, 14]. In combination with the results of [6] it allows us to complete a picture of classical spin systems with ferromagnetic interactions at low temperatures.

If J is any finite range ferromagnetic interaction one associates with it a family $\mathfrak{A}(J)$ of functions on the configuration space and proves that for low enough temperatures all translation invariant equilibrium states agree on elements of this family.

Furthermore, the symmetriy group $\mathscr{S}$ is introduced. It acts on the configuration space of the system by flipping spins at lattice sites in such a way that leaves the energy invariant. Let, for $G\in\mathscr{S}$, ϱ_G^+ be the (equilibrium) state $f\mapsto\varrho^+(f\circ G)$,[1] and for any probability measure μ on $\mathscr{S}$ let

$$\varrho_\mu = \int_{\mathscr{S}} \varrho_G^+ \mu(dG)\,. \tag{0.1}$$

The group $\mathscr{S}(J)$ and the family $\mathfrak{A}(J)$ are closely related: the closed linear span of $\mathfrak{A}(J)$ consists exactly of all $\mathscr{S}$-invariant functions. From the uniqueness on $\mathfrak{A}(J)$ we weduce – this is our main result – that for any ferromagnetic spin system with finite range interaction all $\mathbb{Z}^\nu$-invariant equilibrium states at low enough temperature have the integral representation (0.1).

The representation is made unique by intergrating over $\mathscr{S}/\mathscr{S}^+$ instead of $\mathscr{S}$ where $\mathscr{S}^+$ is the isotropy subgroup of ϱ^+. $\mathbb{Z}^\nu$ acts on $\mathscr{S}/\mathscr{S}^+$ in a natural way, and ϱ_μ is $\mathbb{Z}^\nu$-invariant iff μ is, ϱ_μ is ergodic iff μ is. Thus the description of all invariant equilibrium states at low temperatures is reduced, in a sense, to finding $\mathscr{S}/\mathscr{S}^+$

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[1] ϱ^+ is the equilibrium state defined by fixing the maximal spin outside Λ and letting $\Lambda\nearrow\infty$, Section 2.1.