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Unbounded Derivations of C^*-Algebras II

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Abstract. It is demonstrated that a closed symmetric derivation δ of a $C^\star$-algebra $\mathfrak{A}$ generates a strongly continuous one-parameter group of automorphisms of a $C^\star$-algebra $\mathfrak{A}$ if and only if, it satisfies one of the following three conditions

1. $(\alpha\delta+1)(D(\delta))=\mathfrak{A}$, $\alpha\in\mathbb{R}\backslash\{0\}$.
2. δ possesses a dense set of analytic elements.
3. δ possesses a dense set of geometric elements.

Together with one of the following two conditions

1. $\|(\alpha\delta+1)(A)\|\geqq\|A\|$, $\alpha\in\mathbb{R}$, $A\in D(\delta)$.
2. If $\alpha\in\mathbb{R}$ and $A\in D(\delta)$ then $(\alpha\delta+1)(A)\geqq 0$ implies $A\geqq 0$.

Other characterizations are given in terms of invariant states and the invariance of $D(\delta)$ under the square root operation of positive elements.

1. Introduction

A derivation δ of a $C^\star$-algebra $\mathfrak{A}$ is defined to be a linear mapping from a dense $\star$-subalgebra $D(\delta)\subseteqq\mathfrak{A}$, the domain of δ, to a subspace $R(\delta)\subseteqq\mathfrak{A}$, the range of δ, satisfying the property

$$\delta(AB)=\delta(A)B+A\delta(B), \qquad A, B\in D(\delta).$$

A derivation of this type is called symmetric if

$$\delta(A)^*=\delta(A^*), \qquad A\in D(\delta).$$

A general derivation δ always has a decomposition

$$\delta=\delta_1+i\delta_2$$

in terms of symmetric derivations.

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