Commun. math. Phys. 46, 1—9 (1976)

Communications in Mathematical Physics

Classical KMS Condition and Tomita-Takesaki Theory

G. Gallavotti*

Istituto di Fisica dell'Università di Napoli, Napoli, Italy, and Centre de Physique Théorique, CNRS, Marseille, France

M. Pulvirenti

Istituto Matematico dell'Università di Camerino, Camerino (MA), Italy

Abstract. Relationships between the classical KMS condition and the time evolution for classical systems are discussed.

1. Introduction

The well known theorem of Tomita-Takesaki [1] on automorphisms of Von Neumann algebras becomes trivial in the case of a commutative algebra.

On the other hand the physical ideas related to the Tomita's theorem suggest that there should be some non trivial version of the theorem in the commutative case.

In this paper we analyze a possible "commutative" version of the Tomita-Takesaki's theorem.

2. Non Commutative ("Quantum") Theory

It is believed that states in quantum statistical mechanics can be described by some positive linear functional on an involutive normed algebra $\mathfrak{A}$, with identity.

It is somehow clear that there is not a unique algebra which is useful for the description of a given statistical mechanical system. Usually the algebra $\mathfrak{A}$ is an union of an increasing family of concrete (local) C^*-algebras of bounded operators on Hilbert spaces.

We shall call an algebra $\mathfrak{A}$ of the type just described "algebra of strictly local quantum observables" [2]. We shall drop in what follows, the words "strictly local" when referring to this concept.

A "state" ϱ is a positive normalized linear functional on $\mathfrak{A}$ [2].

Given a state ϱ on $\mathfrak{A}$ we can find a Hilbert space $\mathscr{H}_\varrho$, a representation π of $\mathfrak{A}$ as algebra of bounded operators in $\mathscr{H}_\varrho$, and a cyclic vector $\xi \in \mathscr{H}_\varrho$, such that $\varrho(A) = (\xi, \pi(A)\xi)\ \forall A \in \mathfrak{A}$.

* Postal address: Istituto Matematico dell'Università, Piazzale delle Scienze, I-00185 Roma, Italy.