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Quantized Fields Propagating in Plane-Wave Spacetimes

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Abstract. This paper contains an account of the interaction of a quantized massive scalar field with the classical c number gravitational field of a plane sandwich wave of arbitrary profile and polarization. It is shown that the time varying gravitational field of the wave produces no particles and the Feynman propagator for the problem is calculated exactly. This is used to show that any reasonable regularization of the vacuum expectation value of the energy momentum tensor of the field must vanish. This means that a gravitational wave far from its source will propagate without hindrance by quantum effects.

Introduction

Recently there has been some interest in a class of problems in which one treats a quantized field propagating on a classical, c-number, background gravitational field [1–4]. This work has encountered two sorts of difficulties:

(1) The definition of no particle states.

(2) The definition of a suitable energy momentum tensor operator $\boldsymbol{T}_{\mu\nu}$.

The first difficulty can be circumverted to some extent when the spacetime is asymptotically flat in some sense and interesting results have been obtained [1–3]. The second difficulty remains and becomes especially urgent if one seeks to determine the back reaction of the quantum field on the geometry in a Hartree-Fock approximation of the form

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=8\pi\langle\psi|\boldsymbol{T}_{\mu\nu}|\psi\rangle$$

where $|\psi\rangle$ is some constant Heisenberg state vector – for example the initial no particle or vacuum state $|0_-\rangle$. An especially interesting question is will $\langle 0_-|\boldsymbol{T}_{\mu\nu}|0_-\rangle$ obey the various energy conditions used in the Singularity Theorems or other global results in classical general relativity [6]. The generally expected answer seems to be "no" but no exact calculations have been performed.

This paper contains an exact calculation of $\langle 0_-|\boldsymbol{T}_{\mu\nu}|0_-\rangle$ for the case of a plane sandwich wave of arbitrary profile and polarization in the case that the quantum field is spinless. The main result is that in such a spacetime no particles are produced by the time varying gravitational field and all reasonable suggestions for regularizing the divergent $\langle 0_-|\boldsymbol{T}_{\mu\nu}|0_-\rangle$ give a vanishing result. This shows