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Collision Theory for Waves in Two Dimensions and a Characterization of Models with Trivial S-Matrix

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Abstract. Within the framework of local relativistic quantum theory in two space-time dimensions, we develop a collision theory for waves (the set of vectors corresponding to the eigenvalue zero of the mass operator). Since among these vectors there need not be one-particle states, the asymptotic Hilbert spaces do not in general have Fock structure. However, the definition and "physical interpretation" of an S-matrix is still possible. We show that this S-matrix is trivial if the correlations between localized operators vanish at large timelike distances.

1. Introduction

The aim of the present paper is to develop, within the framework of local relativistic quantum theory in two dimensions, a collision theory for the states corresponding to the eigenvalue zero of the mass operator. It is a peculiarity of the two-dimensional world that the set of these states corresponds in general to a highly reducible representation of the Poincaré group. Moreover, among these states there need not be one-particle states. As a trivial example take the even part of a free, massless theory (the set of even polynomials in the fields acting on the vectors with even particle number). Since the direct product of an arbitrary number of massless one-particle states with positive (or negative) momentum is again a state with mass zero, there are many massless states in such a theory. However, there are no one-particle states.

Therefore, instead of assuming the existence of one-particle states, we take as the basis of our construction the representation spaces $\mathscr{H}_+$ and $\mathscr{H}_-$ of the Poincaré group corresponding to the right and left branch of the light-cone in momentum space[1]. Since the vectors in these spaces are invariant under light-like translations in the $t_+ = (t, t)$ and $t_- = (t, -t)$ direction, respectively, they can be interpreted as excitations *(waves)* moving freely with the speed of light from the left to the right or in the opposite direction. Hence these states are suitable

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[1] Note that the branches of the light-cone in two dimensions are separately invariant under Lorentz transformations. Thus, there exist representations attached to either one of them.