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Interacting Relativistic Boson Fields in the De Sitter Universe with Two Space-Time Dimensions

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Abstract. The positive temperature Gibbs state of a scalar boson field with a relativistic local self-interaction in two space-time dimensional Minkowski universe as constructed in [1] is not relativistic invariant. We prove in this paper that the corresponding state in the De Sitter universe is actually relativistic invariant if the temperature is given by $T=\frac{1}{2\pi R}$ where R is the constant radius of curvature of the De Sitter universe. Moreover the construction gives that the Schwinger functions or imaginary time Wightman functions are the moments of a generalized Markoff process on the sphere of radius R.

0. Introduction

In a recent paper [1], a Markoff field approach has been applied to the study of statistical mechanics for interacting bosons in two space-time dimensions, without cutoff. The Gibbs state at positive temperature is explicitely constructed and it is proved that for every positive temperature the thermodynamic limit exists and is unique for the polynomial and exponential interactions. This is achieved by showing that the Schwinger functions at positive temperature $\frac{1}{\beta}$ $\left(\text{the imaginary time Gibbs states at temperature } \frac{1}{\beta}\right)$ are equal to the Schwinger functions (in the usual terminology) for the interacting field in a periodic box of length β. In turn this is proved expressing the functions S^β by means of a F.K.N.-type formula which links the Gibbs state at temperature $\frac{1}{\beta}$ to a Markoff field on the cylinder $S_\beta \times \mathbb{R}$ (S_β is the circle of length β) which becomes the euclidean Markoff field on $\mathbb{R}^2$ associated with the 0 temperature theory when the curvature of the cylinder goes to 0, i.e. when $\beta \to \infty$.

The lack of relativistic covariance contained in the definition of Gibbs state has, as an imaginary time counter-part, the lack of euclidean covariance of the associated Markoff field and, roughly speaking, is measured by the magnitude of $\frac{1}{\beta}$ or, in other words, by the radius of curvature in time direction of the manifold supporting the associated Markoff field. This suggests the possibility of carrying out the construction of an invariant Gibbs state for a boson scalar field in a finite, homogeneous, isotropic 2-dimensional universe with constant curvature in

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