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Multicomponent Field Theories and Classical Rotators

François Dunlop

Inst. des Hautes Études Scientifiques, F-91 Bures-sur-Yvette, France

Charles M. Newman*

Dept. of Mathematics, Indiana University, Bloomington, Indiana 47401, USA

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Abstract. It is shown that a D-component Euclidean quantum field, $\boldsymbol{\varphi}=(\varphi^1,\ldots,\varphi^D)$, with $\lambda|\boldsymbol{\varphi}|^4+\beta|\boldsymbol{\varphi}|^2$ interaction, can be obtained as a limit of (ferromagnetic) classical rotator models; this extends a result of Simon and Griffiths from the case $D=1$. For these Euclidean field models, it is then shown that a Lee-Yang theorem applies for $D=2$ or 3 and that Griffiths' second inequality is valid for $D=2$; a complete proof is included of a Lee-Yang theorem for plane rotator and classical Heisenberg models. As an application of Griffiths' second inequality for $D=2$, an interesting relation between the "parallel" and "transverse" two-point correlations is obtained.

1. Introduction

Consider a multicomponent scalar field, $\boldsymbol{\varphi}(x,t)=(\varphi^1(x,t),\ldots,\varphi^D(x,t))$, in d-dimensional space-time with Hamiltonian,

$$\int_{\mathbb{R}^{d-1}}\left[\tfrac{1}{2}\textstyle\sum_{i=1}^D((\pi^i)^2+|\nabla\varphi^i|^2+m_0^2(\varphi^i)^2)+\tilde{Q}(\sum_{i=1}^D(\varphi^i)^2)+\sum_{i=1}^D A^i\varphi^i\right]dx\,, \tag{1.1}$$

where $\boldsymbol{\pi}=\partial\boldsymbol{\varphi}/\partial t$, $\tilde{Q}$ is a polynomial with positive highest coefficient, and $\boldsymbol{A}(x,t)$ is an external field interacting with $\boldsymbol{\varphi}$. The problem of constructing a corresponding quantum field has been considerably simplified in recent years by the probabilistic methods of Euclidean field theory (see, for example, the articles in [1]); it has been particularly realized that the associated Euclidean field is closely related (via a "lattice approximation") to certain models of ferromagnets from classical statistical mechanics [2].

The type of model we are concerned with consists of a family of random D-dimensional "spin" vectors $\{\boldsymbol{S}_j=(S_j^1,\ldots,S_j^D): j=1,\ldots,N\}$ with joint probability distribution on $(\mathbb{R}^D)^N$,

$$\frac{1}{Z}\exp\left(\textstyle\sum_{j=1}^N \boldsymbol{a}_j\cdot\boldsymbol{s}_j+\sum_{j,k=1}^N\sum_{i=1}^D J_{jk}^i s_j^i s_k^i\right)\prod_{j=1}^N\varrho_j(\boldsymbol{s}_j)\,, \tag{1.2}$$

where

$$Z=Z(\{\boldsymbol{a}_j\},\{J_{jk}^i\})=\int_{(\mathbb{R}^D)^N}\exp\left(\textstyle\sum_j\boldsymbol{a}_j\cdot\boldsymbol{s}_j+\sum_{j,k,i}J_{jk}^i s_j^i s_k^i\right)\prod_j d\varrho_j(\boldsymbol{s}_j)\,, \tag{1.3}$$

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