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When is a Field Theory a Generalized Free Field?

Klaus Baumann

Bell Laboratories, Murray Hill, New Jersey, USA,
and Max-Planck-Institut für Physik und Astrophysik, München, Federal Republic of Germany

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Abstract. We show within a scalar relativistic quantum field theory that if either some even truncated n-point-function vanishes or some multiple commutator of the field operators is a c-number then the field is necessarily a generalized free field.

I.

The generalized free fields are well known examples for relativistic quantum field theories. Introduced in 1961 by Greenberg [1] they have been extensively studied since then. Of special interest was the question under what conditions a field theory is necessarily a generalized free field. As shown by Dell'Antonio [2], Robinson [3] and Greenberg [4] such a sufficient condition is that the support of the field in momentum space excludes certain domains. Robinson [5] gave another criterion namely the vanishing of the truncated n-point-functions beyond some N. In this note we shall strengthen this result and prove – with entirely different methods than Robinson – that if an arbitrary even truncated Wightman function vanishes the field must be a generalized free field.

II.

We consider a relativistic scalar field $A(x)$ which we assume to fulfill Wightman's axioms [6, 7]. We denote the vacuum state by Ω and the n-point-functions by

$$\mathscr{W}_n(x_1, \ldots, x_n) = (\Omega, A(x_1), \ldots, A(x_n)\Omega).$$

Without restriction we can assume $(\Omega, A(x)\Omega) = 0$. The truncated n-point-functions [7] are recursively defined by

$$\mathscr{W}_1(x) = \mathscr{W}_1^T(x) \quad [\text{and therefore } \mathscr{W}_1^T(x) = 0]$$

$$\begin{aligned}\mathscr{W}_n(x_1, \ldots, x_n) = {} & \textstyle\sum_{\text{partitions}} \mathscr{W}^T_{r_1}(x_{l_1(1)} \ldots x_{l_1(r_1)}) \\ & \times \mathscr{W}^T_{r_2}(x_{l_2(1)} \ldots x_{l_2(r_2)}) \ldots \mathscr{W}^T_{r_s}(x_{l_s(1)} \ldots x_{l_s(r_s)}).\end{aligned}$$

Now we are able to formulate

Theorem 1. *If* $\mathscr{W}^T_{2n}(x_1 \ldots x_{2n}) \equiv 0$ *for some* $n \geqq 2$ *then*

$$[\ldots[A(x_1), A(x_2)] \ldots A(x_n)] = (\Omega, [\ldots[A(x_1), A(x_2)] \ldots A(x_n)]\Omega)^T \cdot \mathbb{1}.$$