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Charges as Integrals over Densities: An Alternative Formulation of Coleman's Theorem

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Abstract. The charge operator is hermitean if and only if the vacuum is invariant. In that case the charge must be invariant under time translations.

Consider within the Wightman framework a quantized real field $t^{\nu m}(x)$ transforming under the inhomogeneous Lorentz group (ILG) according to

$$t^{\mu m}(x) \to U(a,\Lambda) t^{\mu m} U^{-1}(a,\Lambda) = (\Lambda^{-1})^{\mu}_{\mu'} D^m_{m'}(\Lambda^{-1}) t^{\mu' m'}(\Lambda x + a)\,.$$

Here m stands for a collection of vector indices, $U(a,\Lambda)$ is the unitary representation of the ILG, $D^m_{m'}(\Lambda)$ a finite dimensional irreducible representation of the homogeneous Lorentz group (HLG)[1]. Let $\vartheta_r(\vec{x}) = \vartheta\left(\frac{|\vec{x}|}{r}\right)$, $r>0$, $\eta(x)$ be real test functions with compact support and $\vartheta(s)=1$ for $s \leqq 1$, $\int \eta(x^0)\,dx^0 = 1$ (see e.g. [4]). We assume in addition that the Fourier-transform $\tilde{\eta}(p^0) \neq 0$ for all finite p^0 (this is always attainable, e.g., by a small shift in imaginary direction in p°-space). This rather technical looking assumption turns out to be necessary for not loosing in the limit $r \to \infty$ a contribution of $t^{\mu m}$ concentrated on a mass shell (see also [4]).

Put

$$Q^m_r = \int t^{0m}(x) \vartheta_r(\vec{x}) \eta(x^0) d^4x\,,$$

$$\dot{Q}^m_r = \int \partial_0 t^{0m}(x) \vartheta_r(\vec{x}) \eta(x^0) d^4x\,,$$

$$D^m_r = \int \partial_\mu t^{\mu m}(x) \vartheta_r(\vec{x}) \eta(x^0) d^4x\,.$$

Then, due to relative locality,

$$A \to \lim_{r\to\infty} i[Q^m_r, A] = i[Q^m_r, A]_{r \geqq r_0(A)}, \qquad A \in \mathscr{R}$$

defines a map on the algebra $\mathscr{R}$ of all strictly local operators, and defines an operator Q^m by

$$Q^m A\Omega = [Q^m_r, A]|_{r \geqq r_0(A)} \Omega \tag{1}$$

with domain $\mathscr{R}\Omega$ (Ω denotes the unique vacuum vector).

[1] We consider only single-valued representations. Therefore we may assume the $t^{\nu m}$ to be hermitean. Mutatis mutandis the following remains unchanged for complex fields.

[2] This is possible because Ω is separating.