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Resolvents, Semigroups and Gibbs States for Infinite Coupling Constant

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Abstract. The infinite coupling constant limit of the resolvent, the semigroup and the Gibbs state is obtained for a certain class of perturbations.

As an example the infinite intrasite repulsion limit of the one-dimensional Hubbard model with nearest neighbour hopping terms is treated. This system exhibits a phase transition in the thermodynamic limit.

1. Introduction

In contrast to the usual perturbation theory [1] where the behaviour under a "small" perturbation is studied it is the aim of this paper to investigate the influence of "large" perturbations. To be more specific, we shall investigate the behaviour of the resolvent, the semigroup and the Gibbs state associated with $\kappa^{-1}A+B$ as κ tends to zero. The results are contained in Theorem 1 to 3 of Section 2.

In Section 3 the results of Section 2 are applied to the one-dimensional Hubbard model (nearest neighbour hopping) with infinite intrasite repulsion. It is shown that this system may be described as a composition of a system of spinless fermions with a spin system without interaction between the two subsystems. The thermodynamic limit is calculated explicitly exhibiting a phase transition (nonanalyticity at zero chemical potential).

2. General Theory

Let A and B be operators on a Hilbert space $\mathscr{H}$ with the properties

$$A=A^*, \quad B\subset B^*, \quad D(B)\supset D(A). \tag{1}$$

According to the closed graph theorem B is A-bounded, i.e. there exists a constant $\alpha>0$ such that

$$\alpha\|Bu\|\leqq\|Au\|+\|u\| \tag{2}$$

for all $u\in D(A)$. The operator

$$H_\kappa=\kappa^{-1}A+B \tag{3}$$

is self-adjoint on $D(A)$ for real $\kappa\neq0$ with $|\kappa|<\alpha$ [1]. Let $P=E(\{0\})$ where E is the spectral measure associated with A, and $d=\operatorname{dist}(0,\operatorname{supp}E\setminus\{0\})$. As a prefix d means norm ($d>0$) or strong ($d=0$).