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A Local Edge of the Wedge Theorem

Olle Stormark
Department of Mathematics, Royal Institute of Technology, Stockholm, Sweden

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Abstract. A local version of the edge of the wedge theorem is proved. The boundary values are not required to be equal on a whole open neighborhood of the given point, but essentially only along a bunch of lines through this.

1. Introduction

Let g be a hyperfunction defined in a neighborhood U of the origin in $\mathbb{R}^{n+1}$, such that g can be represented as a difference of two boundary values of holomorphic functions:

$$g = \delta_{\Gamma_+}(f_+) - \delta_{\Gamma_-}(f_-),$$

where $f_\pm$ are holomorphic in $U \times i\Gamma_\pm$, $\Gamma_- = -\Gamma_+$ and Γ_+ is a proper open convex cone with vertex in the origin. In this situation the ordinary edge of the wedge theorem says that if $g=0$ on U, then there exists a function f holomorphic in a neighborhood of U in $\mathbb{C}^{n+1}$ such that f is a holomorphic continuation of f_+ and f_-. This follows simply because in that case the hyperfunction $h = \delta_{\Gamma_+}(f_+) = \delta_{\Gamma_-}(f_-)$ has its (decomposed) singular support in the set $U \times i(\Gamma_+^* \cap \Gamma_-^*)$, which is empty. ($\Gamma^*$ denotes the dual cone of Γ.) Hence h is in fact real analytic (cf. [7]).

In this paper we will try to prove a local version of this theorem under somewhat weaker conditions. In order to be able to state these, we first observe that if L is a complex line in $U \times i(\Gamma_+ \cup \Gamma_-)$ then f_+ and f_- by restriction define a hyperfunction $g|_{L\cap\mathbb{R}^{n+1}}$ in one variable. To say that this restriction vanishes means precisely that f_+ and f_- are holomorphic continuations of each other along L (cf. [5]). Our first assumption is then that $g|_{L\cap\mathbb{R}^{n+1}}=0$ for all lines L in $U \times i(\Gamma'_+ \cup \Gamma'_-)$, where $\Gamma'_\pm$ are open subcones of $\Gamma_\pm$. If this is true, $f_\pm$ can be evaluated at the origin of each L, so that we get a function $(f_\pm|_L)(0)$. The second assumption is that this function depends analytically on L. And thirdly we require that the restrictions of all derivatives of g to one certain line in $\Gamma'_+ \cup \Gamma'_-$ vanish. The conclusion is then that there exists a unique common holomorphic continuation of $f_\pm$ to a neighborhood of the origin.

This result is rather similar to the well known Kolm-Nagel theorem (see [4] and [7]), which says that to reach the conclusion it suffices to impose a strengthened form of the third condition above. We have in fact been aiming at finding a hyperfunction version of this theorem (cf. [7], p. 77), but have not been succesful, since the second assumption above is not stated in hyperfunction language and our proof is along very classical lines.

The proof is divided into two steps. By using a theorem on separate analyticity we first get a continuation to a complex cone. Then the continuation theorem of Hartogs can be applied to give the desired conclusion.