Commun. math. Phys. 43, 17—32 (1975)

Holomorphic Versions of the Fabrey-Glimm Representations of the Canonical Commutation Relations*

Judith Kunofsky

Department of Mathematics, University of California, Berkeley, California, USA

Received September 10, 1974; in revised form December 27, 1974

I. Introduction

Glimm and Fabrey have constructed [6; 4] a Hilbert space $\mathscr{F}_r$ for a simplified version of the $:\Phi^4:$ model in quantum field theory for 3 space-time dimensions with space cutoff by using a sequence of truncated exponentials involving $a^*(v)$ to define the dressing transformation, where

$$v(k_1, k_2, k_3, k_4) = \tilde{h}(\Sigma k_i)\, \Pi \mu(k_i)^{-1/2} (\Sigma \mu(k_i))^{-1} .$$

The space cutoff is h, $k_i \in \mathbb{R}^2$, $\mu(k_i) = (\mu_0^2 + |k_i|^2)^{1/2}$. For v of a more general form, lower parameter j, and upper cutoff σ, they show convergence of $(\hat{T}_{j\sigma}\phi, \hat{T}_{j\sigma}\psi)e^{-X(\sigma)}$ for ϕ, ψ in a dense subset $\mathscr{D}$ of Fock space, as $\sigma \to \infty$. $\hat{T}_{j\sigma}$ is a truncated version of $e^{a^*(v)}$ and $X(\sigma)$ is the renormalization. The closure of the inductive limit of $\hat{T}_j \mathscr{D}$ over the lower parameters defines a Hilbert space which carries a Weyl representation of the CCR (canonical commutation relations).

The Bargmann-Segal complex wave representation for the free field has as Hilbert space $H^2(K'_{cx}, d\mu)$, the completion of the tame holomorphic functionals on K'_{cx}, the complex distributions, which are square-integrable with respect to the Gaussian cylinder set measure μ on K'. The finite-dimensional case has been discussed by Bargmann [1] and the infinite dimensional case by Segal [15; 16]. Creation operators on $H^2(K', d\mu)$ are diagonalized and annihilation operators are differentiations.

We construct an analogue to the complex wave representation for the interaction case as a countable inductive limit of spaces of the following form: completion of the tame holomorphic functionals on K'_{cx} in the space of functionals which are square integrable with respect to a countably additive measure associated with T_j. This space carries a representation of the CCR for which creation is a multiplication operator and annihilation is, formally, differentiation plus multiplication by the log derivative of T_j. The representation is unitarily equivalent to the Glimm-Fabrey representation.

For a fixed lower parameter j and upper cutoff σ we construct $H^2(K', d\eta_{j\sigma})$, where $d\eta_{j\sigma} = |T_{j\sigma}|^2 \, \|T_{j\sigma}\|^{-2} d\mu$. In order to show that the $\eta_{j\sigma}$ converge to a countably additive measure, we analyze the characteristic functions $L_{j\sigma}(h)$ of $\eta_{j\sigma}$ and,

* This work was supported in part by the National Science Foundation, GP30798X2.