Commun. math. Phys. 42, 269—279 (1975)

Collision Theory for Massless Fermions

D. Buchholz*
CERN, Geneva, Switzerland

Received December 17, 1974

Abstract. Starting from the basic postulates of local relativistic quantum theory, the asymptotic incoming and outgoing collision states of massless Fermions are constructed. The corresponding Hilbert spaces have Fock structure and thus allow the usual definition of an S-matrix. In contrast to the massive case, there are geometric relations between the local nets of the underlying field algebra and the asymptotic fields.

1. Introduction

In this paper we establish the existence of collision states for massless Fermions in the framework of local relativistic quantum theory. It is amazing that a proof of this fact has not appeared before now – more than ten years after Haag and Ruelle developed their famous collision theory for massive particles [1, 2]. But it might be that their intuitively appealing ideas have turned away the attention of the experts from the simple facts allowing the construction also in the massless case.

The methods of Haag and Ruelle are based on two essential features of massive theories: absence of long-range forces and existence of almost local operators which create one-particle states from the vacuum. These facts make it possible to construct the spaces of incoming and outgoing collision states and to establish their Fock structure[1]. Only in a second step can one then define the asymptotic fields of the particles. But in order to be sure that they act as operators on the whole Hilbert space of states, one needs the additional assumption of asymptotic completeness of the theory.

It is very unlikely that this technique can successfully be carried over to the massless case. Therefore, we apply a completely different method which takes special care of the peculiar kinematics of massless particles. A basic ingredient of our proofs is the trivial fact that these particles move with the speed of light. So they have – loosely speaking – one degree of freedom less in configuration space than their massive counterparts. Imagine, for example, a massless particle which sits at the tip of a light cone in Minkowski space. This particle can never reach interior points of the cone. In fact all interior points of the cone become ultimately space-like to the position of the particle at asymptotic times. This naïve picture may be carried over to quantum theory if the number of space dimensions is odd. It is nothing else but the Huyghens principle [5].

---

* On leave of absence from II. Institut für Theoretische Physik, Universität Hamburg.

[1] Hepp [3] and Herbst [4] observed that actually only one of the above-mentioned properties is needed for a proof.