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Statistical Mechanics of Quantum Lattice Systems without Translation Invariance

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Abstract. The well-known results concerning the equilibrium of a translation invariant quantum lattice system – existence of the pressure and of the time automorphisms, variational principle for the pressure – are generalized to a large class of quantum lattice systems with potentials not exhibiting covariance under the group of lattice translations.

I. Introduction

It is well known [1] that for a quantum lattice system with a suitable interaction $\phi(X)$, $X \subset \mathbb{Z}^\nu$, the pressure $P(\phi)$ and the group of time automorphisms $\tau_t(\phi)$ exist in the thermodynamic limit. In proving these results one makes repeatedly use of the translation covariance of the interaction:

$$\phi(X+a) = \tau_a \phi(X), \tag{1}$$

where $X+a$, $a \in \mathbb{Z}^\nu$, denotes the set obtained by translation of X, and τ_a is the canonical isomorphism of the bounded operators on $\mathscr{H}_X$ onto the bounded operators on $\mathscr{H}_{X+a}$, $\tau_a : B(\mathscr{H}_X) \to B(\mathscr{H}_{X+a})$. $\Big($Our notation is as usual: to each site x of the ν-dimensional lattice $\mathbb{Z}^\nu$ we attribute a copy $\mathscr{H}_x$ of a finite dimensional Hilbert space $\mathscr{H}$, and $\mathscr{H}_X = \bigotimes_{x \in X} \mathscr{H}_x$.$\Big)$ But there are systems for which (1) does not hold: consider, for instance, two-component crystals, crystals with impurities, or systems with inhomogeneous external fields, or even stationary non-equilibrium systems. We shall concentrate here on systems in equilibrium. It will turn out that, due to the fact that our observable algebra is assumed to be quasi-local, $\tau_t(\phi)$ can be defined for all interactions satisfying a temperedness condition which is an obvious generalization of the usual one: There is a norm $|||\phi|||_f$, $f(\xi) = e^{\alpha \xi}$, which has to be finite; if ϕ satisfies (1), $|||\phi|||_f$ conincides with the usual norm $\|\phi\|_f$ which is assumed to be finite in [2], where the existence of $\tau_t(\phi)$ is demonstrated for translation covariant potentials. Furthermore, one can show that $\tau_t(\phi)$ depends continuously on ϕ. These results are contained in Section III. (Precise definitions and notations will be found in the following section.) The existence of the pressure is ensured by a weak form of temperedness, but, in addition, Eq. (1) has to be replaced by a condition which guarantees the existence of a mean of the "local" pressures $P_{\Lambda_0 + x}(\phi)$ for fixed Λ_0. We can deal either with "asymptotically translation covariant potentials" describing a lattice with a locally disturbed potential, or, what is more interesting, treat potentials describing "randomly scattered impurities" in addition to the regular lattice interaction. This class of potentials will turn out to be a fairly large one. This is done in Sections IV–VI.