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Lagrangian Dynamics of Spinning Particles and Polarized Media in General Relativity*

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Abstract. The general form of the Lagrangian equations of motion is derived for a spinning particle having arbitrary multipole structure in arbitrary external fields. It is then shown how these equations, together with the complete system of field equations can be recovered from a fourdimensional action integral representing a polarized dustlike medium interacting with an arbitrary set of fields. These general results are then specialized to the case of Einstein-Maxwell fields in order to obtain the general-relativistic extension of Lorentz's dielectric theory.

I. Introduction

This paper attempts to answer and to trace the connections between two principal questions:

What are the equations of motion of a spinning multipole test particle in given external fields?

For a continuous medium with internal spin and multipole structure, what are the phenomenological field equations relating the fields generated by the medium to its statistical bulk properties? (The classical prototype is Lorentz's dielectric theory.)

Both questions have long histories of research behind them. The close link between them has been stressed in a recent note by one of us [1]: for a gaseous medium in a self-consistent Einstein-Maxwell background field, the field equations are delimited to a virtually unique form by the requirement that they be compatible with conservation laws derived from the equations of motion of the constituent particles. In the present, more complete and more general discussion, we show how the general form of both the equations of motion and the field equations can be derived from a unified Lagrangian viewpoint.

Equations of motion for spinning test bodies have traditionally been derived by two principal methods[1]:

(i) *Multipole Formalism for Extended Body or Equivalent Schwartz Distribution.* This method, due originally to Mathisson [5] and developed further by Papapetrou [6], Taub [7], Dixon [8], Madore [9], Suttorp and de Groot [2], and others [10], proceeds essentially by integrating the conservation identities

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[1] For reviews and extensive bibliographies (especially of the special-relativistic literature), see the books by Suttorp and de Groot [2], Halbwachs [3], and Corben [4].