Commun. math. Phys. 41, 235—266 (1975)

Quantized Fields in Interaction with External Fields

I. Exact Solutions and Perturbative Expansions

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Received November 14, 1973; in revised form October 8, 1974

Abstract. We consider a massive, charged, scalar quantized field interacting with an external classical field. Guided by renormalized perturbation theory we show that whenever the integral equations defining the Feynman or retarded or advanced interaction kernel possess non perturbative solutions, there exists an S-operator which satisfies, up to a phase, the axioms of Bogoliubov, and is given for small external fields by a power series which converges on coherent states. Furthermore this construction is shown to be equivalent to the one based on the Yang-Källen-Feldman equation. This is a consequence of the relations between chronological and retarded Green's functions which are described in detail.

Introduction

Numerous papers have been devoted to the study of the interactions of particles with external fields, within the framework of quantum field theory.

The formal aspects were well developped twenty years ago, in particular through the work of Feynman [1], Matthews and Salam [2] and Schwinger [3]. A good summary can be found e.g. in Thirring's book on Quantum Electrodynamics [4].

Mathematically rigorous non perturbative treatments were given in some particular cases by several authors. Capri [5] has explained lucidly "the reduction to c-number problem" (cf. also Wightman [6]). Bongaarts [7] has treated the case of spin 1/2 particles in a stationary external field. Seiler [8], using the results of a paper by Schroer, Seiler and Swieca [9], proved the existence of a fixed time evolution operator in the following cases: scalar and pure electric external fields for spin 0 and spin 1/2 fields. Wightman [6] has given necessary and sufficient conditions for the existence of a unitary S-operator for arbitrary spins, in the case of external field coupled with quantized fields which fulfil a first order system of partial differential equations.

The perturbative aspects of quantum field theory described for instance in Bogoliubov and Shirkov's classical book [10] have recently been further developped by Epstein and Glaser [11], Steinmann [12] and Zimmermann [13].

These authors have proved the existence of an S-operator as an operator valued formal power series, for arbitrary Wick polynomial Lagrangeans.

The aim of this paper is to exhibit the connection between Bogoliubov's version of perturbation theory and non pertubative methods developped by the above mentioned authors. For the sake of simplicity we shall consider the problem of a charged scalar boson field interacting with a classical external field.