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Existence of the Scattering Matrix for the Linearized Boltzmann Equation

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Abstract. Following Hejtmanek, we consider neutrons in infinite space obeying a linearized Boltzmann equation describing their interaction with matter in some compact set D. We prove existence of the S-matrix and subcriticality of the dynamics in the (weak-coupling) case where the mean free path is larger than the diameter of D uniform in the velocity. We prove existence of the S-matrix also for the case where D is convex and filled with uniformly absorbent material. In an appendix, we present an explicit example where the dynamics is not invertible on L^1_+, the cone of positive elements in L^1.

§ 1. Introduction

In this paper, we consider the linearized Boltzmann equation (LBE):

$$\dot{n}(x, v, t) = -v \cdot \operatorname{grad}_x n(x, v, t) + \int k(x, v', v)\, n(x, v', t)\, dv' - \sigma_a(x, v)\, n(x, v, t)\,. \quad (1)$$

This equation describes a beam of neutrons which is non-self-interacting; thus we are assuming low density and the non-linear term is dropped from the usual Boltzmann equation (3.6). The first term describes free streaming of the beam in phase space, the second the net input in (x, v) phase space due to scattering from other regions (x, v') in phase space and from production (fission!) by other particles. Simiarly, the last term describes loss due to absorbtion and scattering from (x, v) into other regions. We emphasize that while we use the symbol σ_a (in order to have notation similar to Hejtmanek [7]), σ_a is not quite a cross-section but has the units of inverse time and is a rate. $v^{-1}\sigma_a(x, v)$ is the cross-section times the density of scatters (or absorbers) and is an inverse mean free path. Similarly the quantity

$$\sigma_p(x, v) = \int k(x, v, v')\, dv' \quad (2)$$

is a production rate [note that $k(x, v, v')$ appears in (2) but that $k(x, v', v)$ appears in (1)!].

In the cases of greatest physical interest [1, 3, 5, 16], either the configuration space is finite or it is made effectively finite by having pure absorbers ($k = 0$; $\sigma_a(x, v) \geqq \alpha$) arround an interaction region D (so the beam delays exponentially outside D). In some approximations [9, 14, 15] D is taken to be a slab infinite in two directions. In any event, the case we will consider (following Hejtmanek [7]) of free space surrounding a interaction region is quite far from reactor theory. It is nevertheless of some physical interest and is of special interest in the mathematical theory of scattering [10, 12, 19] for two reasons: first the natural data set

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