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Equilibrium States for Classical Systems

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Abstract. It is shown that the Dobrushin-Lanford-Ruelle equations for the probability measure μ, and the Kirkwood-Salsburg type equations for the lattice or continuum correlation functions ϱ, and for the spin correlation functions σ, are essentially equivalent for all ϱ, σ, and μ satisfying certain boundedness conditions. It is also noted that the lattice equations are identical to the equations for the stationary states of a certain Markoff process. This extends previous results of Ruelle, Brascamp and Holley who proved some of these equivalences for states.

1. Introduction

The equilibrium states of an infinite classical lattice system can be specified by various means [1–5], e.g.:

1. States defined by a probability measure μ which is a solution of a linear equation called the "Equilibrium Equation". These states are called "*Equilibrium States*".

2. States defined by the solutions of a set of equations for the correlation function $\varrho(X)$ (lattice gas language).

3. States defined by the solutions of a set of equations for the expectation values $\sigma(X)$ (spin language).

It is known that 1. implies 2. [2, 3, 4] and conversely any *state* whose correlations satisfy 2. is an Equilibrium State. In this note we point out that the Equilibrium Equations and the equations for $\varrho(X)$ or $\sigma(X)$ are equivalent in an even stronger sense, i.e. their solutions, with suitable boundedness properties, even if they do not define states are also equivalent. Indeed the different equations are expressions of the same relation using different basis for the observables.

2. Lattice Systems

We consider a spin $\frac{1}{2}$ classical lattice system defined on a lattice $\mathscr{L}$; the algebra $\mathfrak{A}$ of *observables* is the algebra of continuous functions on the compact set [5]

$$\Gamma = \prod_{x \in \mathscr{L}} \{0, 1\}_x = \mathscr{P}(\mathscr{L}).$$

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