Commun. math. Phys. 40, 235—247 (1975)

The Kirkwood-Salsburg Equations: Solutions and Spectral Properties

Herm Jan Brascamp*

The Institute for Advanced Study, Princeton, New Jersey, USA

Received July 7, 1974

Abstract. It is shown that the Kirkwood-Salsburg equations for a classical lattice gas are equivalent to the Dobrushin-Lanford-Ruelle equilibrium equations. The term "Kirkwood-Salsburg equations" is used here in a restricted sense, and thus the known result for a larger system of equations is improved (see Table 1). Some information on the spectrum of the Kirkwood-Salsburg operator is found in connection with zeros of partition functions. An example is given to show that the Kirkwood-Salsburg equations can have other solutions than states in the space of uniformly bounded correlation functions.

1. Introduction and Notation

Let us first briefly recall some by now standard notation concerning interactions and states for a classical lattice gas (see e.g. [1, 2]). In this paper, we mean by a *lattice* L nothing more than a countable (finite or infinite) number of points. A *configuration* is a subset $X \subset L$, where the points in X are called *occupied*, the points outside X *empty*.

For two sets X, Y with $Y \subset X$, we shall denote the difference by $X - Y$. Further we shall not distinguish in notation between a lattice point x and the set consisting of that point only.

An interaction is a map Φ from the finite, non-empty sets $Y \subset L$ to the real numbers with the property

$$\sum_{Y : x \in Y} |\Phi(Y)| < \infty, \qquad \forall x \in L. \tag{1}$$

The energy of a finite set X, given a fixed configuration $S \subset L - X$, is then

$$W(X \mid S) = \beta \sum_{\substack{Y \subset X \cup S \\ Y \cap X \neq \emptyset}} \Phi(Y) - \beta \mu |X|, \tag{2}$$

where $|X|$ is the number of points in X. For $S = \emptyset$, one usually writes $W(X \mid \emptyset) = U(X)$. The inverse temperature β and the chemical potential μ are absorbed in W for notational convenience.

By Eq. (1), $|W(X \mid S)| < \infty$. Moreover, with $S_M = S \cap M$,

$$\lim_{M \to L - X} W(X \mid S_M) = W(X \mid S) \tag{3}$$

---

* Supported by National Science Foundation Grant GP-40768X.