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Existence of Ground States and KMS States for Approximately Inner Dynamics*

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Abstract. A strongly continuous one parameter group of $*$-automorphisms of a C^*-algebra with unit is said to be approximately inner if it can be approximated strongly by inner one parameter groups of $*$-automorphisms. It is shown that an approximately inner one parameter group of $*$-automorphisms has a ground state and, if there exists a trace state, a KMS state for all inverse temperatures. It follows that quantum lattice systems have ground states and KMS states. Conditions that a strongly continuous one parameter group of $*$-automorphisms of a UHF algebra be approximately inner are given in terms of the unbounded derivation which generates the automorphism group.

Introduction

Suppose $\{\alpha_t; -\infty < t < \infty\}$ is a strongly continuous one parameter group of $*$-automorphisms of a C^*-algebra $\mathfrak{A}$ with unit, where by strongly continuous we mean $\|\alpha_t(A) - A\| \to 0$ as $t \to 0$ for each $A \in \mathfrak{A}$. We say the group $\{\alpha_t\}$ is approximately inner if there exists a sequence $\{H_n\}$ of hermitian elements of $\mathfrak{A}$ such that

$$\|e^{itH_n} A e^{-itH_n} - \alpha_t(A)\| \to 0$$

as $n \to \infty$ for each $A \in \mathfrak{A}$ where for fixed A the convergence is uniform for t in a compact set. In this paper we will show that if $\{\alpha_t\}$ is approximately inner then there exists at least one ground state (Section 2) and there exist KMS states for all inverse temperatures β (Section 3) provided $\mathfrak{A}$ has a trace state. Since for quantum lattice systems the dynamics is given by approximately inner one parameter groups of $*$-automorphisms (see e.g. ([14], p. 193), [13] or [1]) it follows that quantum lattice systems have ground states and KMS states for all inverse temperatures β. Ruelle has shown the existence of ground states for quantum lattice systems in [15, Theorems 2(c) and 4].

In working with a strongly continuous one parameter group of $*$-automorphisms $\{\alpha_t\}$ it is often useful to introduce the unbounded derivation δ which generates the group. Suppose $\{\alpha_t\}$ is a strongly

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