Commun. math. Phys. 38, 257—277 (1974)


# Irreducible Multiplier Corepresentations and Generalized Inducing

Ronald Shaw

Department of Applied Mathematics, University of Hull, Hull, England

Joseph Lever

Department of Tutorial Studies, Langside College, Glasgow, Scotland

Received March 22, 1974

**Abstract.** Wigner's classification of irreducible corepresentations into three types is generalised to irreducible multiplier corepresentations. Representations of Types I, II, and III have commutants isomorphic to $\boldsymbol{R}, \boldsymbol{H}$, and $\boldsymbol{C}$, respectively. The more general problem of relating irreducible multiplier corepresentations of a group to those of an invariant subgroup is considered, and some algebraic aspects of "generalized inducing" are described. The Wigner classification is then re-obtained as a very simple instance of the general theory.

## § 1. Introduction

*1.1.* Consider an irreducible $PUA$-representation (see Parthasarathy [6]) $\boldsymbol{U}$ of a group $G$ with respect to a fixed $UA$-decomposition $G=G^{+}\cup G^{-}$, with $G^{-}$ non-empty. (Thus $G$ must possess an invariant subgroup $G^{+}$ of index 2.) If $U$ is a version of $\boldsymbol{U}$, then $U(g)$ is a unitary ($g\in G^{+}$) or antiunitary ($g\in G^{-}$) operator on a complex Hilbert space $\mathscr{H}$ which satisfies

$$U(g_1)\,U(g_2)=\sigma(g_1,g_2)\,U(g_1g_2)\,, \tag{1.1}$$

where the (generalised) multiplier (with respect to the fixed $UA$-decomposition $G=G^{+}\cup G^{-}$) $\sigma$ is a function $G\times G\rightarrow \boldsymbol{T}$ (=complex numbers of unit modulus) which satisfies, for all $g_1, g_2, g_3\in G$, the equation

$$\sigma(g_1,g_2)\,\sigma(g_1g_2,g_3)=\sigma(g_1,g_2g_3)\,\sigma(g_2,g_3)^{g_1}\,. \tag{1.2}$$

Here $\lambda^{g}(\lambda\in\boldsymbol{C}, g\in G)$ is defined to be $\lambda$ if $g\in G^{+}$ and $\bar{\lambda}$ if $g\in G^{-}$. The map $g\mapsto U(g)$, from $G$ into the unitary antiunitary group of $\mathscr{H}$, is called a multiplier corepresentation of $G$ with multiplier $\sigma$, or simply a $\sigma$-corepresentation. The multiplier $\sigma'$ of any other version $U'$ of $\boldsymbol{U}$ will be equivalent to the multiplier $\sigma$, satisfying that is

$$\sigma'(g_1,g_2)=[\lambda(g_1)\,\lambda(g_2)^{g_1}/\lambda(g_1g_2)]\,\sigma(g_1,g_2) \tag{1.3}$$

for some $\lambda: G\rightarrow \boldsymbol{T}$.